02 POLYNOMIALS

BASIC CONCEPTS & FORMULAE

1. **Polynomial:** An algebraic expression of the form $a_0x^n + a_1x^{n-1} + a_2x^{n-2} + \ldots + a_{n-1}x + a_n$, where $a_0, a_1, a_2, \ldots a_n$ are real numbers, n is a non-negative integer and $a_0 \neq 0$ is called a polynomial of degree n.
2. **Degree of polynomial:** The highest power of variable in a polynomial is called the degree of polynomial.
3. **Types of polynomials:**
   - (*i*) **Constant Polynomial:** A polynomial $p(x)$ of degree zero is called a constant polynomial and it is of the form $p(x) = k$.
   - (*ii*) **Linear Polynomial:** A polynomial of degree one is called linear polynomial and it is of the form $p(x) = ax + b$, where a, b are real numbers and $a \neq 0$.
   - (*iii*) **Quadratic Polynomial:** A polynomial of degree two is called quadratic polynomial and it is of the form $p(x) = ax^2 + bx + c$, where a, b, c are real numbers and $a \neq 0$.
   - (*iv*) **Cubic Polynomial:** A polynomial of degree three is called cubic polynomial and it is of the form $p(x) = ax^3 + bx^2 + cx + d$, where a, b, c, d are real numbers and $a \neq 0$.
   - (*v*) **Bi-quadratic Polynomial:** A polynomial of degree four is called bi-quadratic polynomial and it is of the form $p(x) = ax^4 + bx^3 + cx^2 + dx + e$, where a, b, c, d, e are real numbers and $a \neq 0$.
4. **Graph of polynomial:**
   - (*i*) Graph of a linear polynomial $p(x) = ax + b$ is a straight line.
   - (*ii*) Graph of a quadratic polynomial $p(x) = ax^2 + bx + c$ is a parabola open upwards like $\cup$ if $a > 0$
   - (*iii*) Graph of a quadratic polynomial $p(x) = ax^2 + bx + c$ is a parabola open downwards like $\cap$ if $a < 0$.
   - (*iv*) In general a polynomial $p(x)$ of degree n crosses the x-axis at atmost n points.
5. **Zeroes of a polynomial:** α is said to be zero of a polynomial $p(x)$ if $p(\alpha) = 0$.
   - (*i*) Geometrically, the zeroes of a polynomial $p(x)$ are the x-co-ordinates of the points, where the graph of $y = p(x)$ intersects the x-axis.
   - (*ii*) A polynomial of degree 'n' can have atmost n zeros.

     That is a quadratic polynomial can have atmost 2 zeroes and a cubic polynomial can have atmost 3 zeroes.
   - (*iii*) 0 may a zero of a polynomial.
   - (*iv*) A non-zero constant polynomial have no zeroes.

6. **Discriminant of a quadratic polynomial:** For polynomial $p(x) = ax^2 + bx + c, a \neq 0$, the expression $b^2 - 4ac$ is known as its discriminant 'D'.

$\therefore \quad D = b^2 - 4ac$

7. **Relationship between the zeroes and the coefficients of a polynomial:**

(*i*) If α, β are zeros of $p(x) = ax^2 + bx + c$, then

$$\text{Sum of zeros} = \alpha + \beta = \frac{-b}{a} = \frac{-(\text{Coefficient of } x)}{\text{Coefficient of } x^2}$$

$$\text{Product of zeros} = \alpha\beta = \frac{c}{a} = \frac{\text{Constant term}}{\text{Coefficient of } x^2}$$

(*ii*) If α, β, γ are zeros of $p(x) = ax^2 + bx + cx + d$, then

$$\alpha + \beta + \gamma = \frac{-b}{a} = \frac{-(\text{Coefficient of } x^2)}{\text{Coefficient of } x^3}$$

$$\alpha\beta + \beta\gamma + \gamma\alpha = \frac{c}{a} = \frac{\text{Coefficient of } x}{\text{Coefficient of } x^3}$$

$$\alpha\beta\gamma = \frac{-d}{a} = \frac{-(\text{Constant term})}{\text{Coefficient of } x^3}$$

(*iii*) If $\alpha, \beta,$ are roots of a quadratic polynomial $p(x)$, then

$p(x) = x^2 - (\text{sum of zeroes})x + \text{product of zeroes} \quad \Rightarrow \quad p(x) = x^2 - (\alpha + \beta)x + \alpha\beta$

(*iv*) If α, β, γ are the roots of a cubic polynomial $p(x)$, then

$p(x) = x^3 - (\text{sum of zeroes})\,x^2 + (\text{sum of product of zeroes taken two at a time})\,x - \text{product of zeroes}$

$\Rightarrow \quad p(x) = x^3 - (\alpha + \beta + \gamma)x^2 + (\alpha\beta + \beta\gamma + \gamma\alpha)x - \alpha\beta\gamma$

MULTIPLE CHOICE QUESTIONS

Choose and write the correct option in the following questions.

1. **The zeroes of the polynomial $x^2 - 3x - m(m + 3)$ are** [*CBSE* 2020 (30/2/1)]

(*a*) $m, m + 3$ (*b*) $-m, m + 3$ (*c*) $m, -(m + 3)$ (*d*) $-m, -(m + 3)$

2. **The degree of polynomial having zeroes – 3 and 4 only is** [*CBSE* 2020 (30/5/2)]

(*a*) 2 (*b*) 1 (*c*) more than 3 (*d*) 3

3. **The number of zeroes for a polynomial $p(x)$ where graph of $y = p(x)$ given in Fig. 2.1, is** [*CBSE* 2020 (30/4/1)]

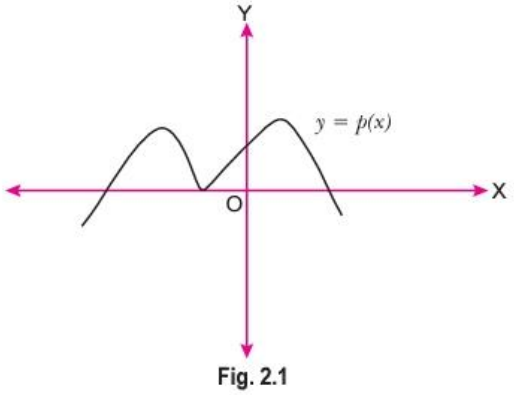

Fig. 2.1

(*a*) 3 (*b*) 4 (*c*) 0 (*d*) 5

4. **In Fig. 2.2, the graph of the polynomial $p(x)$ is given. The number of zeroes of the polynomial is** *[CBSE 2020 (30/3/1)]*

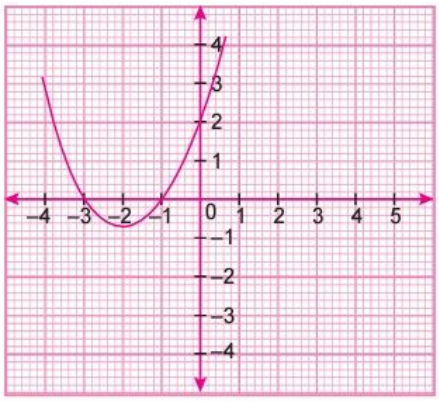

Fig. 2.2

(*a*) 1 (*b*) 2 (*c*) 3 (*d*) 0

5. **The quadratic polynomial, the sum of whose zeroes is – 5 and their product is 6, is** *[CBSE 2020 (30/1/1)]*

(*a*) $x^2 + 5x + 6$ (*b*) $x^2 - 5x + 6$ (*c*) $x^2 - 5x - 6$ (*d*) $-x^2 + 5x + 6$

6. **If one of the zeroes of the quadratic polynomial $x^2 + 3x + k$ is 2, then the value of k is** *[CBSE 2020 (30/1/1)]*

(*a*) 10 (*b*) – 10 (*c*) – 7 (*d*) – 2

7. **The graph of a quadratic polynomial $ax^2 + bx + c = 0$, having discriminant equal to zero, will touch x-axis at exactly how many points?**

(*a*) one (*b*) two (*c*) three (*d*) can't say

8. **The quadratic polynomial $p(x)$ with –24 and 4 as a product and one of the zeros respectively is**

(*a*) $x^2 - 2x - 24$ (*b*) $x^2 + 2x - 24$ (*c*) $x^2 + 2x + 24$ (*d*) Can't be determined

9. **The polynomial $(x - a)$, where $a > 0$, is a factor of the polynomial $q(x) = 4\sqrt{2}x^2 - \sqrt{2}$. Which of these is a polynomial whose factor is $\left(x - \frac{1}{a}\right)$?** *[CBSE Question Bank]*

(*a*) $x^2 + x + 6$ (*b*) $x^2 + x - 6$ (*c*) $x^2 - 5x + 4$ (*d*) $x^2 + 4x - 3$

10. **Which of these is a factor of the polynomial $p(x) = x^3 + 4x + 5$?** *[CBSE Question Bank]*

(*a*) $(x + 1)$ (*b*) $(x - 1)$ (*c*) $(x + 3)$ (*d*) $(x - 3)$

11. **Given that $m + 2$, where m is a positive integer, is a zero of the polynomial $q(x) = x^2 - mx - 6$. Which of these is the value of m?** *[CBSE Question Bank]*

(*a*) 4 (*b*) 3 (*c*) 2 (*d*) 1

12. **Which of these is a zero of the polynomials $p(y) = 3y^3 - 16y - 8$?** *[CBSE Question Bank]*

(*a*) 2 (*b*) 8 (*c*) –2 (*d*) –8

13. **Consider the polynomial in z, $p(z) = z^4 - 2z^3 + 3$. What is the value of the polynomial at $z = -1$?** *[CBSE Question Bank]*

(*a*) 6 (*b*) 5 (*c*) 4 (*d*) 3

14. **Consider the expression $x^{(m^2-1)} + 3x^{\frac{m}{2}}$, where m is a constant. For what value of m, will the expression be a cubic polynomial?** *[CBSE Question Bank]*

(*a*) 1 (*b*) 2 (*c*) –1 (*d*) –2