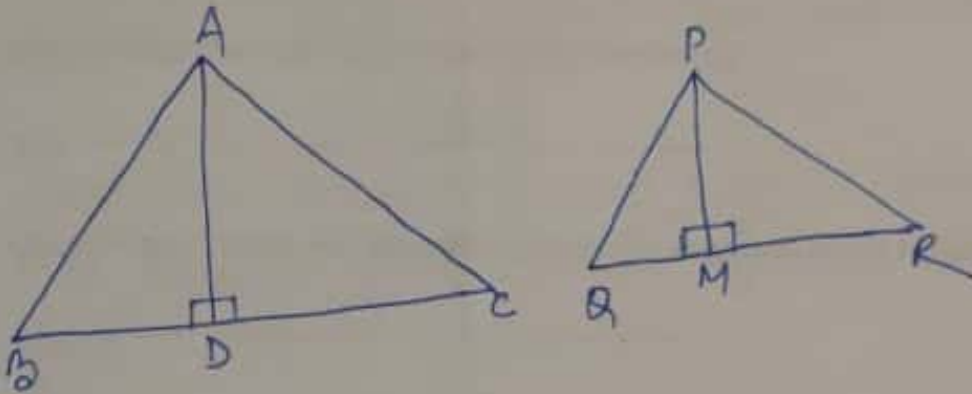


Theorem 6.6

The ratio of the areas of two similar triangles is equal to the square of the ratio of their corresponding sides.



Given :-  $\triangle ABC \sim \triangle PQR$

To Prove :-  $\frac{\text{ar}(\triangle ABC)}{\text{ar}(\triangle PQR)} = \frac{AB^2}{PQ^2} = \frac{BC^2}{QR^2} = \frac{AC^2}{PR^2}$

Const:- Draw  $AD \perp BC$  and  $PM \perp QR$ .

Proof

$\triangle ABD$  and  $\triangle PMQ$

$\angle B = \angle Q$  (Given  $\triangle ABC \sim \triangle PQR$ )

$\angle ADB = \angle PMQ = 90^\circ$

$\triangle ABD \sim \triangle PMQ$  (by A-A)

$$\frac{AB}{PQ} = \frac{AD}{PM} \quad (\text{by C.P.S.T}) \quad \text{--- (i)}$$

$$\frac{AB}{PQ} = \frac{BC}{QR} \quad (\triangle ABC \sim \triangle PQR) \quad \text{--- (ii)}$$

from eqn (i) and (ii)

$$\frac{AD}{PM} = \frac{BC}{QR}$$

$$\frac{\text{ar}(\triangle ABC)}{\text{ar}(\triangle PQR)} = \frac{\frac{1}{2} \times BC \times AD}{\frac{1}{2} \times QR \times PM}$$

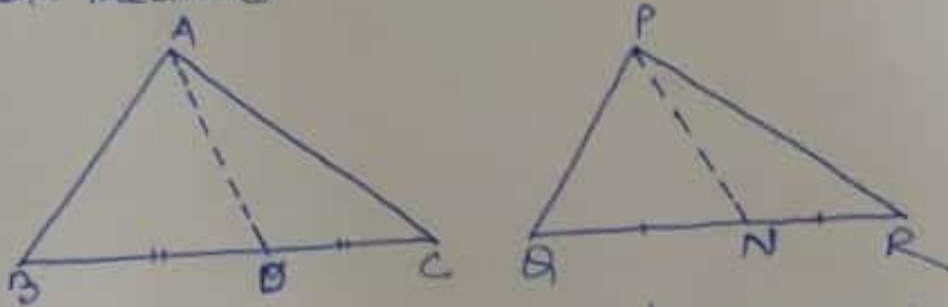
$$= \frac{BC \times AD}{QR \times PM}$$

$$= \frac{BC}{QR} \times \frac{BC}{QR} = \frac{BC^2}{QR^2}$$

$$\text{Hence } \frac{\text{ar}(\triangle ABC)}{\text{ar}(\triangle PQR)} = \frac{AB^2}{PQ^2} = \frac{BC^2}{QR^2} = \frac{AC^2}{PR^2} \quad \underline{\text{Proved}}$$

### Corollary

Prove that the areas of two similar triangles is equal to the square of the ratio of their corresponding ~~sides~~ medians.



Given :- AD and PM are medians of two similar triangles ABC and PQR.

To prove  $\frac{\text{ar}(\triangle ABC)}{\text{ar}(\triangle PQR)} = \frac{AD^2}{PM^2}$

Proof

$\triangle ABD$  and  $\triangle PQN$ .

$\angle B = \angle Q$  ( $\triangle ABC \sim \triangle PQR$ )

$\frac{AB}{PQ} = \frac{BC}{QR}$  ( " )

D and N are mid points of BC and QR

$$\therefore \frac{AB}{PQ} = \frac{BD}{QN}$$

$$\frac{AB}{PQ} = \frac{BD}{QN}$$

$\triangle ABD \sim \triangle PQN$   
(by S-A-S)

$$\frac{AB}{PQ} = \frac{AD}{PN} \quad (\text{by C.P.S.})$$

$$\frac{AB^2}{PQ^2} = \frac{AD^2}{PN^2} \quad (\text{S.S.S})$$

$$\frac{\text{ar}(\triangle ABC)}{\text{ar}(\triangle PQR)} = \frac{AB^2}{PQ^2} \quad \text{--- (i)}$$

(from eq (i) and (ii))

$$\frac{\text{ar}(\triangle ABC)}{\text{ar}(\triangle PQR)} = \frac{AD^2}{PN^2}$$

Proved