Use of differentiation in Physics :→

(i) Instantaneous velocity, $v = \frac{ds}{dt}$

(ii) Instantaneous acceleration, $a = \frac{dv}{dt}$.

(iii) Force applied on a body, $F = ma = m \cdot \frac{dv}{dt}$

(iv) Power, $P = \frac{dw}{dt} = \frac{d(F.S)}{dt} = F \cdot \frac{ds}{dt} = F.v.$

Example :→ A particle is moving with a uniform acceleration, Its displacement at any instant 't' is given by, $S = (10t + 49t^2)$ m What is (i) initial velocity (ii) velocity at $t = 3s$ (iii) uniform acceleration.?

Solⁿ :— A/q, displacement, $S = 10t + 49t^2$.

instantaneous velocity, $v = \frac{ds}{dt}$

ie $v = \frac{d}{dt}(10t + 49t^2)$

or $v = \frac{d}{dt}(10t) + \frac{d}{dt}(49t^2)$

or $v = 10 \times 1t^{1-1} + 49 \times 2t^{2-1}$

or $v = 10 \cdot 1t^0 + 98t$.

ie $v = 10 + 98t$

(i) initial velocity ie velocity at $t = 0$,

$v_i = 10 + 98 \times 0 \therefore v_i = 10 ms^{-1}$.

(ii) velocity at $t = 3s$, $v_3 = 10 + 98 \times 3$

$= 10 + 294 = 304 ms^{-1}$.

(iii) uniform acceleration, $a = \frac{dv}{dt}$

or $a = \frac{d}{dt}(10 + 98t)$

or $a = \frac{d}{dt}(10) + \frac{d}{dt}(98t)$

or $a = 0 + 98 \times 1t^{1-1}$

$\therefore a = 98 \times 1 = 98 \, m/s^2$

Use of integration in Physics:—

A particle is moving with uniform acceleration,

$$a = (2t^2 + 3t + 4.)\ m s^{-2}$$

Find its initial velocity and distance covered in $t = 2s$.

Sol^n:- A/q, $a = (2t^2 + 3t + 4)\ m s^{-2}$

i.e $\frac{dv}{dt} = 2t^2 + 3t + 4$

or $dv = (2t^2 + 3t + 4)\,dt$

On integrating both sides,

$\int dv = \int (2t^2 + 3t + 4)\,dt$

or $\int dv = \int 2t^2 dt + \int 3t\,dt + \int 4\,dt.$

or $V = 2\left[\frac{t^{2+1}}{2+1}\right] + 3\left[\frac{t^{1+1}}{1+1}\right] + 4[t]$

or $V = \frac{2}{3}t^3 + \frac{3}{2}t^2 + 4t.$

Initial velocity, i.e at $t = 0$,

$V_i = \frac{2}{3}(0)^3 + \frac{3}{2}(0)^2 + 4(0) \therefore V_i = 0.$

If you are asked to find velocity at $= 1s$ then

$V_1 = \frac{2}{3}(1)^3 + \frac{3}{2}(1)^2 + 4 \cdot 1$

or $V_1 = \frac{2}{3} + \frac{3}{2} + 4 = \frac{4 + 9 + 24}{6} = \frac{37}{6}$ m/s.

As we know that

$V = \frac{ds}{dt}$ or $\frac{ds}{dt} = \frac{2}{3}t^3 + \frac{3}{2}t^2 + 4t$

or $ds = \left(\frac{2}{3}t^3 + \frac{3}{2}t^2 + 4t\right)dt$

On integrating both sides.

$\int ds = \int \frac{2}{3}t^3 dt + \int \frac{3}{2}t^2 dt + \int 4t\,dt$

or $S = \frac{2}{3}\left(\frac{t^{3+1}}{3+1}\right) + \frac{3}{2}\left(\frac{t^{2+1}}{2+1}\right) + 4\left[\frac{t^{1+1}}{1+1}\right]$

or $S = \frac{2}{3 \times 4}t^4 + \frac{2}{2 \times 3}t^3 + \frac{4}{2}t^2 = \frac{1}{6}t^4 + \frac{1}{3}t^3 + 2t^2$

distance covered in $t = 2s$; $S = \frac{1}{6}(2)^4 + \frac{1}{3}(2)^3 + 2(2)^2 = \frac{16}{6} + \frac{8}{3} + 8$

$\therefore S = \frac{16 + 16 + 48}{6} = \frac{80}{6} = \frac{40}{3}$ m

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