(3) $v^2 = u^2 + 2as$ (Calculus Proof)

As we know that,

$$a = \frac{dv}{dt} = \frac{dv}{dt} \cdot \frac{ds}{dt}$$

or $a = \frac{dv}{ds} \cdot v$ or $v\,dv = a\,ds$

On integrating both sides,

$$\int_u^v v\,dv = \int_0^s a\,ds \quad \text{or} \quad \left[\frac{v^2}{2}\right]_u^v = a\left[s\right]_0^s$$

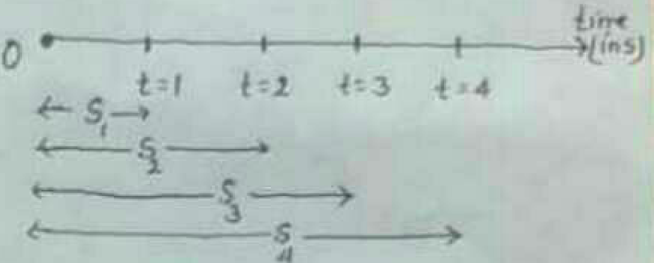
or $\frac{1}{2}\left[v^2 - u^2\right] = a\left[s - 0\right]$ or $v^2 - u^2 = 2as$

$\therefore v^2 = u^2 + 2as$ Prooved.

(4) distance covered in t^{th} s,

$$S_{t^{th}} = u + \left(\frac{2t-1}{2}\right)a \rightarrow \text{Calculus Proof:}$$

It is very clear that



distance Covered in 3rd Sec,

= distance covered in 3 s

— distance covered in 2 s

ie $S_{3rd} = S_3 - S_2$

Similarly, $S_{4th} = S_4 - S_3$, Thus we can write

$$S_{t^{th}} = S_t - S_{t-1}$$

or $S_{t^{th}} = \int_0^t ds - \int_0^{t-1} ds$ Since $\int ds = s$

or $S_{t^{th}} = \cancel{\int_0^{t-1} ds} + \int_{t-1}^{t} ds - \cancel{\int_0^{t-1} ds}$

or $S_{t^{th}} = \int_{t-1}^{t} ds$ or $S_{t^{th}} = \int_{t-1}^{t} v\,dt$ $\left[v = \frac{ds}{dt}\right.$ $ds = v\,dt$

or $S_{t^{th}} = \int_{t-1}^{t} (u + at)\,dt$ $v = u + at$

or $S_{t^{th}} = \int_{t-1}^{t} u\,dt + \int_{t-1}^{t} at\,dt = u\left[t\right]_{t-1}^{t} + a\left[\frac{t^2}{2}\right]_{t-1}^{t}$

or $S_{t^{th}} = u\left[t - (t-1)\right] + \frac{a}{2}\left[t^2 - (t-1)^2\right]$ or $S_{t^{th}} = u\left[\cancel{t} - \cancel{t} + 1\right] + \frac{a}{2}\left[\cancel{t^2} - \cancel{t^2} + 2t - 1\right]$

$\therefore S_{t^{th}} = u + \left(\frac{2t-1}{2}\right)a$ Proved.