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• Vector Product or Cross Product of two vectors

The vector product or cross product of two vectors $\vec{A}$ and $\vec{B}$ is another vector $\vec{C}$, whose magnitude is equal to the product of the magnitudes of the two vectors and sine of the smaller angle between them.

If θ is the smaller angle between $\vec{A}$ and $\vec{B}$, then

$$\vec{A} \times \vec{B} = \vec{C} = AB \sin\theta \hat{C}$$

where $\hat{C}$ is a unit vector in the direction of $\vec{C}$. The direction of $\vec{C}$ or $\hat{C}$ (*i.e.*, vector product of two vectors) is perpendicular to the plane containing $\vec{A}$ and $\vec{B}$ and pointing in the direction of advance of a right handed screw when rotated from $\vec{A}$ to $\vec{B}$.

- Some important properties of cross-product are as follows:
  - (*a*) For parallel as well as anti-parallel vectors (*i.e.*, when $\theta = 0°$ or $180°$), the cross-product is zero.
  - (*b*) The magnitude of cross-product of two perpendicular vectors is equal to the product of the magnitudes of the given vectors.
  - (*c*) Vector product is anti-commutative *i.e.*, $\vec{A} \times \vec{B} = -\vec{B} \times \vec{A}$
  - (*d*) Vector product is distributive *i.e.*, $\vec{A} \times (\vec{B} + \vec{C}) = \vec{A} \times \vec{B} + \vec{A} \times \vec{C}$
  - (*e*) $\vec{A} \times \vec{B}$ does not change sign under reflection *i.e.*, $(-\vec{A}) \times (-\vec{B}) = \vec{A} \times \vec{B}$
  - (*f*) For unit orthogonal vectors, we have

    $$\hat{i} \times \hat{i} = \hat{j} \times \hat{j} = \hat{k} \times \hat{k} = 0, \quad \hat{i} \times \hat{j} = \hat{k}, \quad \hat{j} \times \hat{k} = \hat{i} \quad \text{and} \quad \hat{k} \times \hat{i} = \hat{j}$$

    Moreover $\quad \hat{j} \times \hat{i} = -\hat{k}, \quad \hat{k} \times \hat{j} = -\hat{i} \quad \text{and} \quad \hat{i} \times \hat{k} = -\hat{j}$
  - (*g*) In terms of components $\vec{A} \times \vec{B} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ A_x & A_y & A_z \\ B_x & B_y & B_z \end{vmatrix}$.

- The angular velocity of a body or a particle is defined as the ratio of the angular displacement of the body or the particle to the time interval during which this displacement occurs.

$$\omega = \frac{d\theta}{dt}$$

The direction of angular velocity is along the axis of rotation. It is measured in radian/sec and its dimensional formula is $[M^0L^0T^{-1}]$.

The relation between angular velocity and linear velocity is given by

$$\vec{v} = \vec{\omega} \times \vec{r}$$

- The angular acceleration of a body is defined as the ratio of the change in the angular velocity to the time interval.

$$\text{Angular acceleration} = \frac{\text{Change in angular velocity}}{\text{time taken}}$$

$$\vec{\alpha} = \frac{d\vec{\omega}}{dt}$$

The unit of angular acceleration is rad s^{-2} and dimensional formula is $[M^0L^0L^{-2}]$.

• Torque

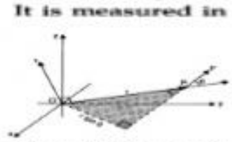
Torque is the moment of force. Torque acting on a particle is defined as the product of the magnitude of the force acting on the particle and the perpendicular distance of the application of force from the axis of rotation of the particle.

Torque or moment of force = force × perpendicular distance

$$\vec{\tau} = \vec{r} \times \vec{F} = rF \sin\theta \, \hat{n}$$

where θ is smaller angle between $\vec{r}$ and $\vec{F}$; $\hat{n}$ is unit vector along $\vec{\tau}$.

It is measured in Nm and has dimensions of $[ML^2T^{-2}]$.



• Angular Momentum

The angular momentum (or moment of momentum) about an axis of rotation is a vector quantity, whose magnitude is equal to the product of the magnitude of momentum and the perpendicular distance of the line of action of momentum from the axis of rotation and its direction is perpendicular to the plane containing the momentum and the perpendicular distance.

It is given by

$$\vec{L} = \vec{r} \times \vec{p}$$

SI unit of angular momentum is kg m^2s^{-1} and its dimensional formula is $[M^1L^2T^{-1}]$.

- Geometrically, the angular momentum of a particle is equal to twice the product of its mass and the areal velocity, *i.e.*,

$$L = 2\text{ m} \times \frac{dA}{dt}$$

- Torque (τ) and angular momentum are corelated as:

$$\vec{\tau} = \frac{d\vec{L}}{dt}$$

- If no net external torque acts on a system then the total angular momentum of the system remains conserved. Mathematically, if $\vec{\tau}_{ext} = \vec{0}$, then

$$\vec{L} = \vec{L}_1 + \vec{L}_2 + \ldots\ldots + \vec{L}_n = \text{a constant}$$

• Axis of Rotation

A rigid body is said to be rotating if every point mass that makes it up, describes a circular path of a different radius but the same angular speed. The circular paths of all the point masses have a common centre. A line passing through this common centre is the axis of rotation.

• A rigid body is said to be in equilibrium if under the action of forces/torques, the body remains in its position of rest or of uniform motion.

For translational equilibrium, the vector sum of all the forces acting on a body must be zero. For rotational equilibrium, the vector sum of torques of all the forces acting on that body about the reference point must be zero. For complete equilibrium, both these conditions must be fulfilled.

• Couple

Two equal and opposite forces acting on a body but having different lines of action, form a couple. The net force due to a couple is zero, but they exert a torque and produce rotational motion.

• Moment of Inertia

The rotational inertia of a rigid body is referred to as its moment of inertia.

The moment of inertia of a body about an axis is defined as the sum of the products of the masses of the particles constituting the body and the square of their respective perpendicular distance from the axis.

It is given by .

$$I = m_1r_1^2 + m_2r_2^2 + \ldots\ldots\ldots\ldots + m_nr_n^2 = \sum_{i=1}^{n} m_ir_i^2,$$

where m_i is the mass and r_i the distance of the i^{th} particle of the rigid body from the axis of rotation. It is measured in kg m^2 and has the dimension of $[ML^2]$.