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Class 11Sc Sub-Physics(Unit-6) Dt- 04 01 2021

• Radius of Gyration

The distance of a point in a body from the axis of rotation, at which if whole of the mass of the body were supposed to be concentrated, its moment of inertia about the axis of rotation would be the same as that determined by the actual distribution of mass of the body is called radius of gyration.

If we consider that the whole mass of the body is concentrated at a distance K from the axis of rotation, then moment of inertia I can be expressed as $I = MK^2$

where M is the total mass of the body and K is the radius of gyration. It is given as

$$K = \sqrt{\left(\frac{r_1^2 + r_2^2 + ... + r_n^2}{n}\right)}$$

• Theorem of Parallel Axes

According to this theorem, the moment of inertia I of a body about any axis is equal to its moment of inertia about a parallel axis through centre of mass, I_{cm}, plus Ma^2 where M is the mass of the body and V is the perpendicular distance between the axes, i.e.,

$I = I_{cm} + Ma^2$

• Theorem of Perpendicular Axes

According to this theorem, the moment of inertia I of the body about a perpendicular axis is equal to the sum of moments of inertia of the body about two axes at right angles to each other in the plane of the body and intersecting at a point where the perpendicular axis passes, i.e.,

$$I = I_x + I_y$$

- A body in rotatory motion possesses rotational kinetic energy given by:

$$\text{Rotational } K.E. = \frac{1}{2} I \omega^2.$$

- In terms of moment of inertia of a body, its angular momentum is defined as the product of moment of inertia and angular velocity *i.e.*,

$$\vec{L} = I\vec{\omega}$$

- Torque may be defined as the produce of moment of inertia and the angular acceleration *i.e.*,

$$\vec{\tau} = I\vec{\alpha}$$

• Rolling Motion

The combination of rotational motion and the translational motion of a rigid body is known as rolling motion.

The kinetic energy associated with a body rolling is the sum of the translational and rotational kinetic energies, *i.e.*, K.E of rolling $= \frac{1}{2}Mv^2 + \frac{1}{2}I\omega^2$

- When a body rolls down an inclined plane (θ) without slipping, the velocity on reaching the ground is,

$$v = \sqrt{\frac{2gh}{1 + \frac{K^2}{r^2}}},$$

where h is the vertical height of inclined plane and K is the radius of gyration of the rolling body.

- The acceleration of a body rolling down an inclined plane is given by

$$a = \frac{g \sin\theta}{\left(1 + \frac{K^2}{r^2}\right)}.$$

• Law of Conservation of Angular Momentum

According to the law of conservation of angular momentum, if there is no external couple acting, the total angular momentum of a rigid body or a system of particles is conserved.

If the moment of inertia of the body changes from I_1 to I_2 due to the change of the distribution of mass of the body, then angular velocity of the body changes from $\vec{\omega}_1$ to $\vec{\omega}_2$, such that

$$I_1\vec{\omega}_1 = I_2\vec{\omega}_1 \quad \text{or} \quad I_1\omega_1 = I_2\omega_2.$$