V. B. Balika Vidyapith, Lakhisarai.

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Relation between rotational kinetic energy and moment of inertia. As shown in Fig. 7.36, consider a rigid body rotating about an axis OZ with uniform angular velocity ω. The body may be assumed to consist of n particles of masses $m_1, m_2, m_3, \ldots, m_n$; situated at distances $r_1, r_2, r_3, \ldots, r_n$ from the axis of rotation. As the angular velocity ω of all the n particles is same, so their linear velocities are

$$v_1 = r_1 \omega, \; v_2 = r_2 \omega, \; v_3 = r_3 \omega, \ldots, \; v_n = r_n \omega$$

Hence the total kinetic energy of rotation of the body about the axis OZ is

Rotational K.E.

$$= \frac{1}{2} m_1 v_1^2 + \frac{1}{2} m_2 v_2^2 + \frac{1}{2} m_3 v_3^2 + \ldots + \frac{1}{2} m_n v_n^2$$

$$= \frac{1}{2} m_1 r_1^2 \omega^2 + \frac{1}{2} m_2 r_2^2 \omega^2 + \frac{1}{2} m_3 r_3^2 \omega^2 + \ldots + \frac{1}{2} m_n r_n^2 \omega^2$$

$$= \frac{1}{2} (m_1 r_1^2 + m_2 r_2^2 + m_3 r_3^2 + \ldots + m_n r_n^2) \omega^2$$

$$= \frac{1}{2} (\Sigma \, m r^2) \omega^2$$

But $\Sigma \, m r^2 = I$, the moment of inertia of the body about the axis of rotation.

$\therefore$ Rotational K.E. $= \frac{1}{2} I \omega^2$

When $\omega = 1$, rotational K.E. $= \frac{1}{2} I$

or $\quad I = 2 \times$ Rotational K.E.

Hence the moment of inertia of a rigid body about an axis of rotation is numerically equal to twice the rotational kinetic energy of the body when it is rotating with unit angular velocity about that axis.