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Subject:-Mathematics

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Find $\lim_{x\to 0} f(x)$ and $\lim_{x\to 1} f(x)$, where $f(x) = \begin{cases} 2x+3, & x \le 0 \\ 3(x+1), & x > 0 \end{cases}$

The given function is

$$f(x) = \begin{cases} 2x+3, & x \le 0 \\ 3(x+1), & x > 0 \end{cases}$$

$$\lim_{x\to 0^-} f(x) = \lim_{x\to 0}[2x+3] = 2(0)+3 = 3$$

$$\lim_{x\to 0^+} f(x) = \lim_{x\to 0} 3(x+1) = 3(0+1) = 3$$

$$\therefore \lim_{x\to 0^-} f(x) = \lim_{x\to 0^+} f(x) = \lim_{x\to 0} f(x) = 3$$

$$\lim_{x\to 1^-} f(x) = \lim_{x\to 1} 3(x+1) = 3(1+1) = 6$$

$$\lim_{x\to 1^+} f(x) = \lim_{x\to 1} 3(x+1) = 3(1+1) = 6$$

$$\therefore \lim_{x\to 1^-} f(x) = \lim_{x\to 1^+} f(x) = \lim_{x\to 1} f(x) = 6$$

Question:-

Evaluate $\lim_{x\to 0} f(x)$, where $f(x) = \begin{cases} \dfrac{|x|}{x}, & x \neq 0 \\ 0, & x = 0 \end{cases}$

Solution:-

$$f(x) = \begin{cases} \dfrac{|x|}{x}, & x \neq 0 \\ 0, & x = 0 \end{cases}$$

$$\lim_{x \to 0^-} f(x) = \lim_{x \to 0^-} \left[\frac{|x|}{x} \right]$$

$$= \lim_{x \to 0} \left(\frac{-x}{x} \right) \qquad \left[\text{When } x \text{ is negaitve, } |x| = -x \right]$$

$$= \lim_{x \to 0} (-1)$$

$$= -1$$

$$\lim_{x \to 0^+} f(x) = \lim_{x \to 0^+} \left[\frac{|x|}{x} \right]$$

$$= \lim_{x \to 0} \left[\frac{x}{x} \right] \qquad \left[\text{When } x \text{ is positive, } |x| = x \right]$$

$$= \lim_{x \to 0} (1)$$

$$= 1$$

It is observed that $\lim_{x \to 0^-} f(x) \neq \lim_{x \to 0^+} f(x)$.

Hence, $\lim_{x \to 0} f(x)$ does not exist.