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Subject:-Mathematics

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Solve this-----

Evaluate the Given limit: $\lim_{x\to 0}\frac{\sin ax+bx}{ax+\sin bx}\ a,b,a+b\neq 0$

At $x=0$, the value of the given function takes the form $\frac{0}{0}$.

Now,

$$\lim_{x\to 0}\frac{\sin ax+bx}{ax+\sin bx}$$

$$=\lim_{x\to 0}\frac{\left(\frac{\sin ax}{ax}\right)ax+bx}{ax+bx\left(\frac{\sin bx}{bx}\right)}$$

$$=\frac{\left(\lim_{ax\to 0}\frac{\sin ax}{ax}\right)\times\lim_{x\to 0}(ax)+\lim_{x\to 0}bx}{\lim_{x\to 0}ax+\lim_{x\to 0}bx\left(\lim_{bx\to 0}\frac{\sin bx}{bx}\right)} \qquad [\text{As } x\to 0\Rightarrow ax\to 0 \text{ and } bx\to 0]$$

$$=\frac{\lim_{x\to 0}(ax)+\lim_{x\to 0}bx}{\lim_{x\to 0}ax+\lim_{x\to 0}bx} \qquad \left[\lim_{x\to 0}\frac{\sin x}{x}=1\right]$$

$$=\frac{\lim_{x\to 0}(ax+bx)}{\lim_{x\to 0}(ax+bx)}$$

$$=\lim_{x\to 0}(1)$$

$$=1$$

Evaluate the Given limit: $\lim_{x\to 0}\left(\operatorname{cosec} x-\cot x\right)$

At $x=0$, the value of the given function takes the form $\infty-\infty$.

Now,

$$\lim_{x\to 0}\left(\operatorname{cosec} x-\cot x\right)$$

$$=\lim_{x\to 0}\left(\frac{1}{\sin x}-\frac{\cos x}{\sin x}\right)$$

$$=\lim_{x\to 0}\left(\frac{1-\cos x}{\sin x}\right)$$

$$=\lim_{x\to 0}\frac{\left(\frac{1-\cos x}{x}\right)}{\left(\frac{\sin x}{x}\right)}$$

$$=\frac{\lim_{x\to 0}\frac{1-\cos x}{x}}{\lim_{x\to 0}\frac{\sin x}{x}}$$

$$=\frac{0}{1}\qquad\left[\lim_{x\to 0}\frac{1-\cos x}{x}=0 \text{ and } \lim_{x\to 0}\frac{\sin x}{x}=1\right]$$

$$=0$$