10|09|xx Class- XII Sc (MATHS) K. Kanhaiya

Topic :- Integration :-

(1) $\int \frac{dx}{\sqrt{x^2-a^2}}$

Let $x = a\sec\theta$, $\frac{dx}{d\theta} = a\sec\theta\tan\theta$

$\therefore dx = a\sec\theta\tan\theta d\theta$

$$a\int \frac{\sec\theta\tan\theta\, d\theta}{\sqrt{a^2\sec^2\theta - a^2}}$$

$$a\int \frac{\sec\theta\tan\theta\, d\theta}{a\sqrt{\sec^2\theta - 1}}$$

$$\int \frac{\sec\theta\tan\theta\, d\theta}{\tan\theta}$$

$$\int \sec\theta d\theta = \log|\sec\theta - \tan\theta| + C_1$$

$$\Rightarrow \log\left|\frac{x}{a} + \sqrt{\frac{x^2}{a^2} - 1}\right| + C_1$$

$$\Rightarrow \log\left|\frac{x + \sqrt{x^2-a^2}}{a}\right| + C_1$$

$$\Rightarrow \log|x + \sqrt{x^2-a^2}| - \log|a| + C_1$$

$$\Rightarrow \log|x + \sqrt{x^2-a^2}| + C \quad \text{where } C = C_1 - \log|a|$$

(2) $\int \frac{dx}{\sqrt{a^2-x^2}} = \sin^{-1}\frac{x}{a} + C$

Let $x = a\sin\theta$

$\frac{dx}{d\theta} = a\cos\theta \Rightarrow dx = a\cos\theta\, d\theta$

$$\int \frac{a\cos\theta\, d\theta}{\sqrt{a^2 - a^2\sin^2\theta}} = \int \frac{a\cos\theta\, d\theta}{a\sqrt{1-\sin^2\theta}}$$

$$\int \frac{\cos\theta}{\cos\theta} d\theta = \int d\theta = \theta + C$$

$$\sin^{-1}\frac{x}{a} + C.$$

(3) $\int \frac{dx}{\sqrt{x^2+a^2}}$?

Let $x = a\tan\theta$, $\frac{dx}{d\theta} = a\sec^2\theta\, d\theta$

$dx = a\sec^2\theta\, d\theta$

$$\int \frac{a\sec^2\theta\, d\theta}{\sqrt{a^2\tan^2\theta + a^2}} = \int \frac{a\sec^2\theta}{a\sec\theta}\, d\theta$$

$$\int \sec\theta\, d\theta = \log|\sec\theta + \tan\theta| + C_1$$

$\Rightarrow \log\left|x + \sqrt{x^2+a^2}\right| + C$.