

Methods of Integration :-

- (a) Integration by Substitution.
- (b) Integration using partial fraction
- (c) Integration by parts.

Now, let us start by substitution :-

(1) $\int \sin mx \, dx$ where $t = x$

let $mx = t$

$$\frac{dt}{dx} = m \Rightarrow \frac{dt}{m} = dx$$

$$\int \frac{\sin t \, dt}{m}$$

$$\frac{1}{m} \int \sin t \, dt = \frac{1}{m} (-\cos t) + C$$

$$\Rightarrow \boxed{-\frac{\cos mx + C}{m}}$$

$$2) \int 2x \sin(x^2+1) dx$$

$$\text{Let } x^2+1 = t$$

differentiating w.r.t x

$$\frac{dt}{dx} = 2x$$

$$\frac{dt}{2} = x dx$$

$$\int \cancel{2} \sin t \frac{dt}{\cancel{2}} = \int \sin t dt$$

$$\Rightarrow -\cos t + C \Rightarrow -\cos(x^2+1) + C.$$

~~Differentiate~~ ^{Integrate} $y = \sin(\tan^{-1}x)$ w.r.t x
 $\frac{1}{1+x^2}$

$$\int \frac{\sin(\tan^{-1}x)}{1+x^2} dx =$$

$$\text{Let } \tan^{-1}x = t \quad \therefore \frac{dt}{dx} = \frac{1}{1+x^2}$$

$$\therefore dt = \frac{dx}{x^2+1}$$

$$\int \sin t dt = -\cos t + C$$
$$= -\cos(\tan^{-1}x) + C$$