

08/09/xx

Class-XII^{sc} (MATHS) K. KanhaiyaTopic \Rightarrow Integrals :-

$$1) \int \frac{dx}{x^2 - a^2}$$

$$\int \frac{dx}{(x+a)(x-a)}$$

$$\int \left[\frac{1}{x+a} - \frac{1}{x-a} \right] dx$$

$$-\frac{1}{2a} \int \frac{1}{x+a} dx + \frac{1}{2a} \int \frac{dx}{x-a}$$

$$-\frac{1}{2a} \log|x+a| + \frac{1}{2a} \log|x-a|$$

$$\frac{1}{2a} \log \left| \frac{x-a}{x+a} \right| + c$$

$$11) \int \frac{dx}{a^2 - x^2} = \int \frac{dx}{(a+x)(a-x)}$$

$$\frac{1}{2a} \int \left[\frac{1}{a+x} + \frac{1}{a-x} \right] dx$$

$$\frac{1}{2a} \left[\log|a+x| + \log|a-x| (-1) \right]$$

$$\Rightarrow \frac{1}{2a} [\log |a+x| - \log |a-x|]$$

$$= \frac{1}{2a} \log \left| \frac{a+x}{a-x} \right| + \underline{\underline{c}}$$

$$(iii) \int \frac{dx}{x^2+a^2}$$

$$\text{let } x = a \tan \theta$$

$$\frac{dx}{d\theta} = a \sec^2 \theta$$

$$dx = a \sec^2 \theta d\theta$$

$$\int \frac{a \sec^2 \theta d\theta}{a^2 \tan^2 \theta + a^2} = \int \frac{a \sec^2 \theta}{a^2 (1 + \tan^2 \theta)} d\theta$$

$$\int \frac{a \sec^2 \theta}{a^2 \sec^2 \theta} d\theta$$

$$\frac{1}{a} \int d\theta = \frac{1}{a} \theta + c$$

$$= \boxed{\frac{1}{a} \tan^{-1} \frac{x}{a} + c}$$