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Class-XII (MATHS)

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Integration by Partial fractions.

If $\frac{P(x)}{Q(x)}$ be a rational function

then it is said to be proper if degree of $P(x)$ is less than the degree of $Q(x)$.

Otherwise this rational function is called improper function.

Note: Improper function can be reduced to proper function by dividing it by long division method.

Ex. $\frac{P(x)}{Q(x)} = T(x) + \frac{P_1(x)}{Q(x)}$

Form of the Rational function

Form of the partial

1) $\frac{Px+q}{(x-a)(x-b)}$

$$\frac{A}{x-a} + \frac{B}{x-b}$$

2) $\frac{Px+q}{(x-a)^2}$

$$\frac{A}{x-a} + \frac{B}{(x-a)^2}$$

$$3) \frac{Px^2 + 9x + 8}{(x-a)(x-b)(x-c)} = \frac{A}{x-a} + \frac{B}{x-b} + \frac{C}{x-c}$$

$$4) \frac{Px^2 + 9x + 8}{(x-a)(x^2 + bx + c)} = \frac{A}{(x-a)} + \frac{Bx + C}{x^2 + bx + c}$$

Ex. 1 Find $\int \frac{dx}{(x+1)(x+2)}$

$$\frac{1}{(x+1)(x+2)} = \frac{A}{x+1} + \frac{B}{x+2}$$

$$1 = A(x+2) + B(x+1)$$

At $x = -2$

$$1 = B(-2+1)$$

$$1 = -1B$$

$$\boxed{-1 = B}$$

At $x = -1$

$$1 = A(-1+2) + 0$$

$$\boxed{1 = A}$$

$$\therefore \int \frac{dx}{(x+1)(x+2)} = \int \frac{1}{x+1} dx - \int \frac{dx}{x+2}$$

$$= \log|x+1| - \log|x+2|$$

$$= \log \left| \frac{x+1}{x+2} \right| + C$$