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Class - XII^{sc} (MATHS)

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Topic \Rightarrow Integration \Rightarrow

$$\Rightarrow \int \frac{(3 \sin \phi - 2) \cos \phi}{5 - \cos^2 \phi - 4 \sin \phi} d\phi.$$

~~let~~
$$\int \frac{(3 \sin \phi - 2) \cos \phi}{5 - (1 - \sin^2 \phi) - 4 \sin \phi} d\phi$$

$$\int \frac{(3 \sin \phi - 2) \cos \phi}{5 - 1 + \sin^2 \phi - 4 \sin \phi} d\phi$$

$$\int \frac{(3 \sin \phi - 2) \cos \phi}{\sin^2 \phi - 4 \sin \phi + 4} d\phi$$

Let $\sin \phi = t$

diff w.r.t ϕ
$$\frac{dt}{d\phi} = \cos \phi$$

$$dt = \cos \phi d\phi$$

$$\int \frac{(3t - 2) dt}{t^2 - 4t + 4}$$

$$\int \frac{(3t - 2) dt}{(t - 2)^2}$$

$$\int \frac{(3t-2)}{(t-2)^2} dt$$

$$\frac{3t-2}{(t-2)^2} = \frac{A}{t-2} + \frac{B}{(t-2)^2}$$

$$3t-2 = A(t-2) + B$$

$$3t-2 = At - 2A + B$$

By equating

$$A = 3, \quad -2A + B = -2$$

$$\text{At } A = 3$$

$$-2 \times 3 + B = -2 \Rightarrow B = -2 + 6 \\ \Rightarrow B = 4.$$

$$\int \left[\frac{3}{t-2} + \frac{4}{(t-2)^2} \right] dt$$

$$\int \frac{3}{t-2} dt + \int \frac{4}{(t-2)^2} dt$$

$$3 \log|t-2| + 4 \frac{(t-2)^{-1}}{-1} + C$$

$$3 \log|t-2| - \frac{4}{(t-2)} + C$$

$$\therefore \boxed{3 \log|\sin\phi - 2| - \frac{4}{(\sin\phi - 2)} + C}$$