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4. A manufacturer makes two types of toys A and B. Three machines are needed for this purpose and the time (in minutes) required for each toy on the machines is give below:

Types of Toys	Machines		
	I	II	III
A	12	18	6
B	6	0	9

Each machine is available for a maximum of 6 hours per day. If the profit on each toy of type A is Rs 7.50 and that on each toy of type B is Rs 5, show that 15 toys of type A and 30 of type B should be manufactured in a day to get maximum profit.

Solution:

Let x and y toys of type A and type B be manufactured in a day respectively.

The given problem can be formulated as given below

Maximize $z = 7.5x + 5y$ ………….. (i)

Subject to the constraints,

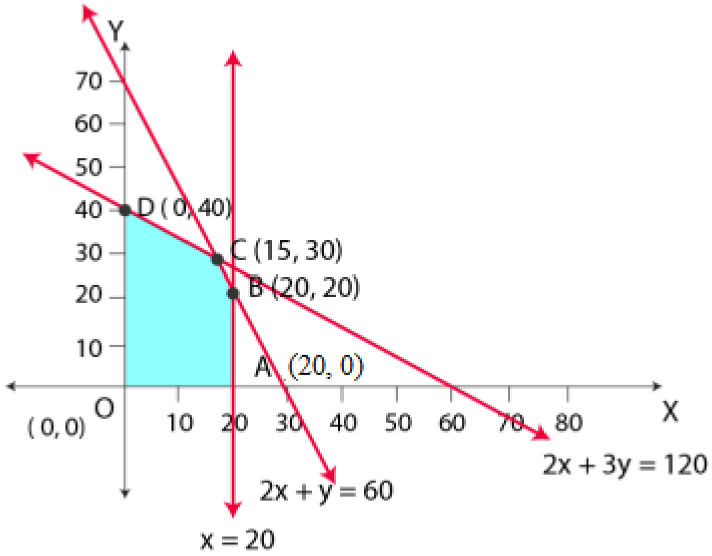
$2x + y \leq 60$ …………. (ii)

$x \leq 20$ ……….. (iii)

$2x + 3y \leq 120$ ……….. (iv)

$x, y \geq 0$ ……………. (v)

The feasible region determined by the constraints is given below



A (20, 0), B (20, 20), C (15, 30) and D (0, 40) are the corner points of the feasible region.

The values of z at these corner points are given below

Corner point	z = 7.5x + 5y	
A (20, 0)	150	
B (20, 20)	250	
C (15, 30)	262.5	Maximum
D (0, 40)	200	

262.5 at (15, 30) is the maximum value of z

Hence, the manufacturer should manufacture 15 toys of type A and 30 toys of type B to maximize the profit.

5. An aeroplane can carry a maximum of 200 passengers. A profit of Rs 1000 is made on each executive class ticket and a profit of Rs 600 is made on each economy class ticket. The airline reserves at least 20 seats for executive class. However, at least 4 times as many passengers prefer to travel by economy class than by the executive class. Determine

how many tickets of each type must be sold in order to maximize the profit for the airline. What is the maximum profit?

Solution:

Let the airline sell x tickets of executive class and y tickets of economy class respectively.

The mathematical formulation of the given problem can be written as given below

Maximize $z = 1000x + 600y$ ………… (i)

Subject to the constraints,

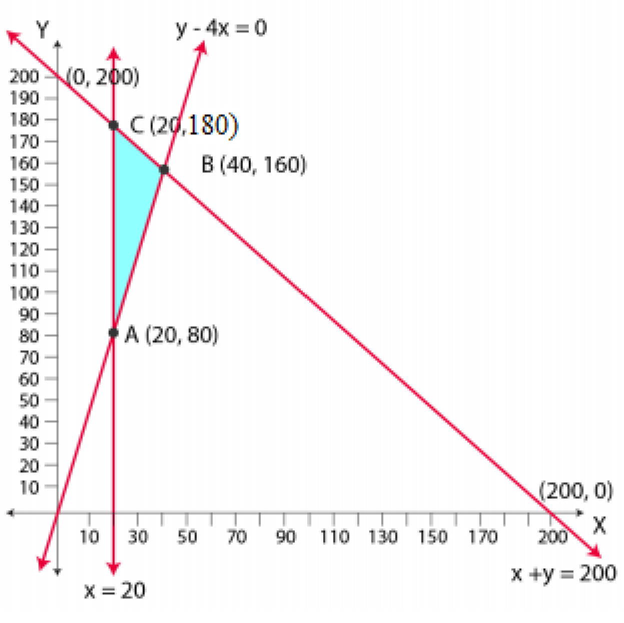
$x + y$ ≤ 200 …………….. (ii)

$x \geq 20$ ………… (iii)

$y - 4x \geq 0$ …………… (iv)

$x, y \geq 0$ ……………(v)

The feasible region determined by the constraints is given below



A (20, 80), B (40, 160) and C (20, 180) are the corner points of the feasible region

The values of z at these corner points are given below

Corner point	$z = 1000x + 600y$	

A (20, 80)	68000	
B (40, 160)	136000	Maximum
C (20, 180)	128000	

136000 at (40, 160) is the maximum value of z

Therefore, 40 tickets of executive class and 160 tickets of economy class should be sold to maximize the profit and the maximum profit is Rs 136000.

6. Two godowns A and B have grain capacity of 100 quintals and 50 quintals respectively. They supply to 3 ration shops, D, E and F whose requirements are 60, 50 and 40 quintals respectively. The cost of transportation per quintal from the godowns to the shops are given in the following table:

Transportation cost per quintal (in Rs)		
From / To	**A**	**B**
D	**6**	**4**
E	**3**	**2**
F	**2.50**	**3**

How should the supplies be transported in order that the transportation cost is minimum? What is the minimum cost?

Solution:

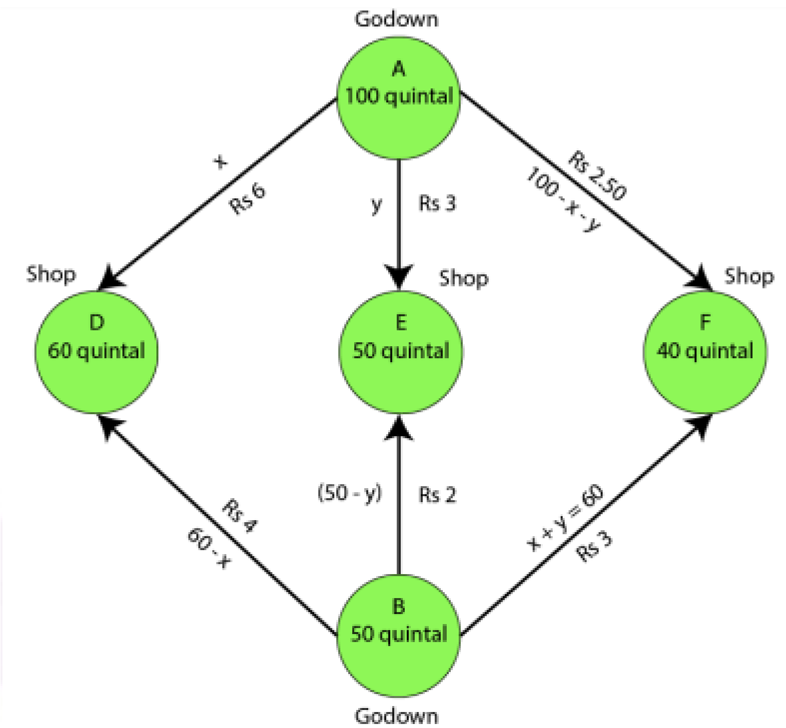
Let godown A supply x and y quintals of grain to the shops D and E

So, (100 – x – y) will be supplied to shop F.

Since, x quintals are transported from godown A, so the requirement at shop D is 60 quintals. Hence, the remaining (60 – x) quintals will be transported from godown B.

Similarly, (50 – y) quintals and 40 – (100 – x – y) = (x + y – 60) quintals will be transported from godown B to shop E and F

The given problem can be represented diagrammatically as given below



$x \geq 0$, $y \geq 0$, and $100 - x - y \geq 0$

Then, $x \geq 0$, $y \geq 0$, and $x + y \leq 100$

$60 - x \geq 0$, $50 - y \geq 0$, and $x + y - 60 \geq 0$

Then, $x \leq 60$, $y \leq 50$, and $x + y \geq 60$

Total transportation cost z is given by,

$z = 6x + 3y + 2.5 (100 - x - y) + 4 (60 - x) + 2 (50 - y) + 3 (x + y - 60)$

$= 6x + 3y + 250 - 2.5x - 2.5y + 240 - 4x + 100 - 2y + 3x + 3y - 180$

$= 2.5x + 1.5y + 410$

The given problem can be formulated as given below

Minimize $z = 2.5x + 1.5y + 410$ …………. (i)

Subject to the constraints,

x + y ≤ 100 ……….. (ii)

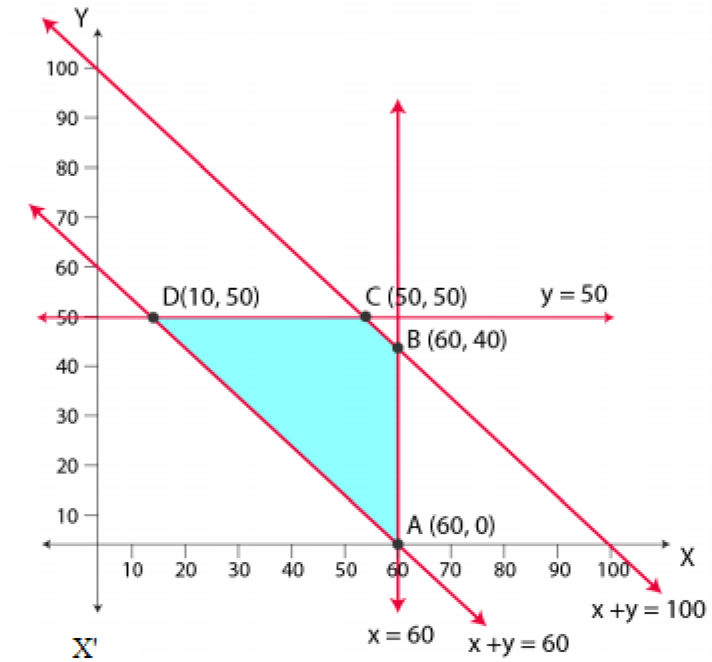
x ≤ 60 ……….. (iii)

y ≤ 50 ………. (iv)

x + y ≥ 60 ……… (v)

x, y ≥ 0 …………..(vi)

The feasible region determined by the system of constraints is given below



A (60, 0), B (60, 40), C (50, 50) and D (10, 50) are the corner points

The values of z at these corner points are given below

Corner point	z = 2.5x + 1.5y + 410	
A (60, 0)	560	
B (60, 40)	620	
C (50, 50)	610	

D (10, 50)	510	Minimum

The minimum value of z is 510 at (10, 50)

Hence, the amount of grain transported from A to D, E and F is 10 quintals, 50 quintals and 40 quintals respectively and from B to D, E and F is 50 quintals, 0 quintals, 0 quintals respectively

Thus, the minimum cost is Rs 510