VIDYA BHAWAN BALIKA VIDYA PITH

शक्तिउत्थानआश्रमलखीसरायबिहार

Class :-12(Maths)   Date:-28.12.2020

1. Show that the line joining the origin to the point (2, 1, 1) is perpendicular to the line determined by the points (3, 5, –1), (4, 3, –1).

Solution:

Let us consider OA be the line joining the origin (0, 0, 0) and the point A (2, 1, 1).

And let BC be the line joining the points B (3, 5, −1) and C (4, 3, −1)

So the direction ratios of OA = $(a_1, b_1, c_1) \equiv [(2 – 0), (1 – 0), (1 – 0)] \equiv (2, 1, 1)$

And the direction ratios of BC = $(a_2, b_2, c_2) \equiv [(4 – 3), (3 – 5), (-1 + 1)] \equiv (1, -2, 0)$

Given:

OA is ⊥ to BC

Now we have to prove that:

$a_1a_2 + b_1b_2 + c_1c_2 = 0$

Let us consider LHS: $a_1a_2 + b_1b_2 + c_1c_2$

$a_1a_2 + b_1b_2 + c_1c_2 = 2 \times 1 + 1 \times (-2) + 1 \times 0$

$= 2 – 2$

$= 0$

We know that R.H.S is 0

So LHS = RHS

∴ OA is ⊥ to BC

Hence proved.

2. If l_1, m_1, n_1 and l_2, m_2, n_2 are the direction cosines of two mutually perpendicular lines, show that the direction cosines of the line perpendicular to both of these are $(m_1n_2 – m_2n_1)$, $(n_1l_2 – n_2l_1)$, $(l_1m_2 – l_2m_1)$

Solution:

Let us consider l, m, n be the direction cosines of the line perpendicular to each of the given lines.

Then, $ll_1 + mm_1 + nn_1 = 0$ … (1)

And $ll_2 + mm_2 + nn_2 = 0$ … (2)

Upon solving (1) and (2) by using cross – multiplication, we get

$$\frac{l}{m_1n_2 - m_2n_1} = \frac{m}{n_1l_2 - n_2l_1} = \frac{n}{l_1m_2 - l_2m_1}$$

Thus, the direction cosines of the given line are proportional to

$(m_1n_2 – m_2n_1), (n_1l_2 – n_2l_1), (l_1m_2 – l_2m_1)$

So, its direction cosines are

$$\frac{m_1n_2 - m_2n_1}{\lambda}, \frac{n_1l_2 - n_2l_1}{\lambda}, \frac{l_1m_2 - l_2m_1}{\lambda}$$

Where,

$$\lambda = \sqrt{(m_1n_2 - m_2n_1)^2 + (n_1l_2 - n_2l_1)^2 + (l_1m_2 - l_2m_1)^2}$$

Where,

$$\lambda = \sqrt{(m_1n_2 - m_2n_1)^2 + (n_1l_2 - n_2l_1)^2 + (l_1m_2 - l_2m_1)^2}$$

We know that

$(l_1^2 + m_1^2 + n_1^2)(l_2^2 + m_2^2 + n_2^2) – (l_1l_2 + m_1m_2 + n_1n_2)^2$

$= (m_1n_2 – m_2n_1)^2 + (n_1l_2 – n_2l_1)^2 + (l_1m_2 – l_2m_1)^2$ … (3)

It is given that the given lines are perpendicular to each other.

So, $l_1l_2 + m_1m_2 + n_1n_2 = 0$

Also, we have

$l_1^2 + m_1^2 + n_1^2 = 1$

And, $l_2^2 + m_2^2 + n_2^2 = 1$

Substituting these values in equation (3), we get

$(m_1n_2 – m_2n_1)^2 + (n_1l_2 – n_2l_1)^2 + (l_1m_2 – l_2m_1)^2 = 1$

$\lambda = 1$

Hence, the direction cosines of the given line are $(m_1n_2 – m_2n_1)$, $(n_1l_2 – n_2l_1)$, $(l_1m_2 – l_2m_1)$

3. Find the angle between the lines whose direction ratios are a, b, c and b – c, c – a, a – b.

Solution:

Angle between the lines with direction ratios a_1, b_1, c_1 and a_2, b_2, c_2 is given by

$$\cos\theta = \left|\frac{a_1a_2 + b_1b_2 + c_1c_2}{\sqrt{a_1^2 + b_1^2 + c_1^2}\sqrt{a_2^2 + b_2^2 + c_2^2}}\right|$$

Given:

$a_1 = a$, $b_1 = b$, $c_1 = c$

$a_2 = b – c$, $b_2 = c – a$, $c_2 = a – b$

Let us substitute the values in the above equation we get,

$$\cos\theta = \left|\frac{a(b-c)+b(c-a)+c(a-b)}{\sqrt{a^2+b^2+c^2}\sqrt{(b-c)^2+(c-a)^2+(a-b)^2}}\right|$$

= 0

Cos θ = 0

So, θ = 90° [Since, cos 90 = 0]

Hence, Angle between the given pair of lines is 90°.