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Class 12Sc   Sub Physics (Unit 06)   Date 09 09 2020

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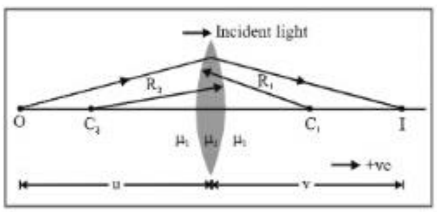
6.4 Lens maker's formula and lens formula

Consider an object O placed at a distance u from a convex lens as shown in figure. Let its image I after two refractions from spherical surfaces of radii R_1 (positive) and R_2 (negative) be formed at a distance v from the lens. Let v_1 be the distance of image formed by refraction from the refracting surface of radius R_1. This image acts as an object for the second surface. Using,



$$\frac{\mu_2}{v}-\frac{\mu_1}{u}=\frac{\mu_2-\mu_1}{R}$$ twice, we have

or $$\frac{\mu_2}{v_1}-\frac{\mu_1}{u}=\frac{\mu_2-\mu_1}{R_1} \quad ...(i)$$

and $$\frac{\mu_1}{v}-\frac{\mu_2}{v_1}=\frac{\mu_1-\mu_2}{R_2} \quad ...(ii)$$

Adding eqs. (i) and (ii) and then simplifying, we get

$$\frac{1}{v}-\frac{1}{u}=\left(\frac{\mu_2}{\mu_1}-1\right)\left(\frac{1}{R_1}-\frac{1}{R_2}\right) \quad ...(iii)$$

This expression relates the image distance v of the image formed by a thin lens to the object distance u and to the thin lens properties (index of refraction and radii of curvature). It is valid only for paraxial rays and only when the lens thickness is much less then R_1 and R_2. The focal length f of a thin lens is the image distance that corresponds to an object at infinity. So, putting $u = \infty$ and $v = f$ in the above equation, we have

$$\frac{1}{f}=\left(\frac{\mu_2}{\mu_1}-1\right)\left(\frac{1}{R_1}-\frac{1}{R_2}\right) \quad ...(iv)$$

If the refractive index of the material of the lens is μ and it is placed in air, $\mu_2 = \mu$ and $\mu_1 = 1$ so that eq. (iv) becomes

$$\frac{1}{f}=(\mu-1)\left(\frac{1}{R_1}-\frac{1}{R_2}\right) \quad ...(v)$$

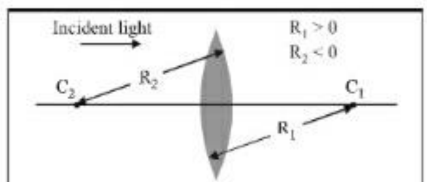
This is called the lens maker's formula because it can be used to determine the values of R_1 and R_2 that are needed for a given refractive index and a desired focal length f.

Combining eqs. (iii) and (v), we get

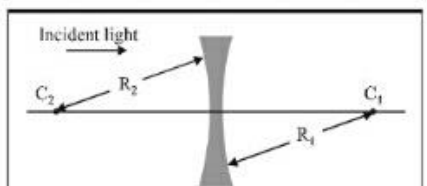
$$\frac{1}{v}-\frac{1}{u}=\frac{1}{f} \quad ...(vi)$$

Which is known as the lens formula. Following conclusions can be drawn from eqs. (iv), (v) and (vi).

1. For a converging lens, R_1 is positive and R_2 is negative. Therefore, $\left(\frac{1}{R_1}-\frac{1}{R_2}\right)$ in eq. (v) comes out a positive quantity and if the lens is placed in air, $(\mu - 1)$ is also a positive quantity. Hence, the focal length f of a converging lens comes out to be positive. For a diverging lens however, R_1 is negative and R_2 is positive and the focal length f becomes negative.



(a)



(b)

2. Focal length of a mirror ($f_M = R/2$) depends only upon the radius of curvature R while that of a lens [eq. (iv)] depends on μ_1, μ_2, R_1 and R_2. Thus, if a lens and a mirror are immersed in some liquid, the focal length of lens would change while that of the mirror will remain unchanged.

3. Suppose $\mu_2 < \mu_1$ in eq. (iv), i.e., refractive index of the medium (in which lens is placed) is more than the refractive index of the material of the lens, then $\left(\frac{\mu_2}{\mu_1}-1\right)$ becomes a negative quantity, i.e., the lens changes its behaviour. A converging lens behaves as a diverging lens and vice-versa. An air bubble in water seems as a convex lens but