17/10/xx Class-XII^{sc} (MATHS) K. Kanhaiya

Topic :- Homogeneous differential eq^n

A function $f(x, y)$ is said to be homogeneous function of degree n if $f(\lambda x, \lambda y) = \lambda^n f(x, y)$ for any non-zero constant λ.

A differential eq^n of the form $\frac{dy}{dx} = f(x, y)$ is said to be homogeneous if $f(x, y)$ is a homogeneous of degree zero

i) Show that the differential eq^n $(x-y)\frac{dy}{dx} = x + 2y$ is a homogeneous and solve it

Ans→ The given differential eq^n can be expressed as

$$\frac{dy}{dx} = \frac{x+2y}{x-y}$$

Let $f(x, y) = \dfrac{x + 2y}{x - y}$

$$f(\lambda x, \lambda y) = \lambda^0 \left(\frac{x + 2\lambda}{x - \lambda y}\right)$$

$$= \lambda^0 . f(x, y)$$

$\therefore$ $f(x, y)$ is a homogenous function of degree zero. So the given differential eq^n is a homogenous differential eq^n.

Let $y = vx$

diff w.r.t x

$$\frac{dy}{dx} = v + x\frac{dv}{dx}$$

Substituting the value of y and $\frac{dy}{dx}$ in eq^n (1) we get

$$v + x\frac{dv}{dx} = \frac{1 + 2v}{1 - v}$$

$$x\frac{dv}{dx} = \frac{1 + 2v}{1 - v} - v$$

$$x\frac{dv}{dx} = \frac{1 + 2v - v + v^2}{1 - v}$$

$$\frac{v-1}{-v^2-v-1}\,dv = \frac{dx}{x}$$

$$\int \frac{v-1}{-v^2-v-1}\,dv = \int \frac{dx}{x}$$

(5) Now integrate yourself using partial fraction.

So, we get desired answer

Scanned by CamScanner