

Deriving a formula for compound interest  
or

Derivation of compound interest.

$$C.I = P \left\{ \left( 1 + \frac{r}{100} \right)^n - 1 \right\}$$

Let  $P$  is the sum on which interest is compounded annually at a rate of  $r\%$  per annum.

$$S.I \text{ of 1st year} = \frac{P \times R \times T}{100} = \frac{P \times r \times 1}{100} = \frac{Pr}{100}$$

$$\begin{aligned} \text{Amount of last first year} &= P + S.I \\ &= P + \frac{Pr}{100} = P \left( 1 + \frac{r}{100} \right) \end{aligned}$$

$$\text{let } y = P \left( 1 + \frac{r}{100} \right) \quad \text{--- (1)}$$

Again

$$S.I \text{ of 2nd year} = \frac{y \times r \times 1}{100} = \frac{yr}{100}$$

$$\begin{aligned} \text{Amount of last 2nd year} &= y + \frac{yr}{100} \\ &= y \left( 1 + \frac{r}{100} \right) \end{aligned}$$

Putting the value of  $y$

$$\begin{aligned} \text{Amount} &= P \left( 1 + \frac{r}{100} \right) \left( 1 + \frac{r}{100} \right) \text{ from eqn (1)} \\ &= P \left( 1 + \frac{r}{100} \right)^2 \end{aligned}$$

Proceeding in this way the amount at the end of  $n$  years will be

$$\text{Amount} = P \left( 1 + \frac{r}{100} \right)^n$$

$$C.I = \text{Amount} - \text{Principal}$$

$$= P \left(1 + \frac{r}{100}\right)^n - P$$

$$C.I = P \left\{ \left(1 + \frac{r}{100}\right)^n - 1 \right\}$$

Calculate the amount and compound interest on.

Principal = ₹ 10800,  $n = 3$  years, Rate =  $12\frac{1}{2}\%$  per annum compounded annually

Sol<sup>n</sup>

$$\text{Amount} = P \left(1 + \frac{r}{100}\right)^n$$

$$P = ₹ 10800,$$

$$n = 3 \text{ years}$$

$$r = 12\frac{1}{2}\% = \frac{25}{2}\%$$

$$\text{Amount} = 10800 \left(1 + \frac{25}{2 \times 100}\right)^3$$

$$= 10800 \left(1 + \frac{25}{2 \times 100}\right)^3$$

$$= 10800 \left(1 + \frac{1}{8}\right)^3$$

$$= 10800 \left(\frac{9+1}{8}\right)^3$$

$$= 10800 \left(\frac{9}{8}\right)^3$$

$$= 10800 \times \frac{9}{8} \times \frac{9}{8} \times \frac{9}{8}$$

$$= \frac{675 \times 729}{32} = \frac{492075}{32}$$

$$= 15377.34$$



$$\text{Amount} = ₹15377.34 \text{ Ans.}$$

$$\text{Compound Interest} = \text{Amount} - \text{Principal}$$

$$= ₹15377.34 - ₹10800$$

$$= ₹4577.34 \text{ Ans.}$$

Do your self

Calculate the amount and compound interest on

(i) ₹ 18000 for 2 ~~1~~ years at 10% per annum compounded annually.

(ii) ₹ 62500 for 1  $\frac{1}{2}$  years at 8% per annum compounded half yearly.

(iii) ₹ 8000 for 1 year at 9% per annum compounded half yearly.

(iv) ₹ 10000 for 1 year at 8% per annum compounded half yearly.

— x — x — x — x —