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Subject: Mathematics

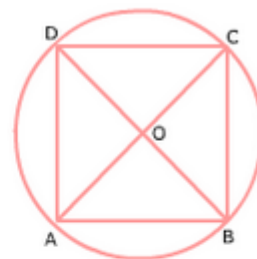
Class-IX

ST:-Prabhat Ranjan

Solve this question-----

If diagonals of a cyclic quadrilateral are diameters of the circle through the vertices of the quadrilateral, prove that it is a rectangle.

Solution:



In $\triangle OAB$ and $\triangle OCD$,

$OA = OC$ | Radii of a circle

$OB = OD$ | Radii of a circle

$\angle AOB = \angle COD$ | Vertically Opposite Angles

$\triangle OAB \cong \triangle OCD$ | SAS Rule

$AB = CD$ | CPCT

$\Rightarrow \text{arc } AB = \text{arc } CD$..(1)

Similarly, we can show that

$\text{arc } AD = \text{arc } CB$ (2)

Adding (1) and (2), we get

$\text{Arc } AB + \text{Arc } AD = \text{Arc } CD + \text{Arc } CB$

$\Rightarrow \text{Arc } BAD = \text{Arc } BCD$

\Rightarrow BD divides the circle into two equal parts (each a semicircle)

$\therefore \angle A = 90^\circ, \angle C = 90^\circ$ | Angle of a semi-circle is 90°

Similarly, we can show that

$\angle B = 90^\circ, \angle D = 90^\circ$

$\angle A = \angle B = \angle C = \angle D = 90^\circ$

\therefore ABCD is a rectangle.

